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On the quantization constraints for a D3-brane in the geometry of NS5 branes

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Abstract: A D3-brane in the background of NS5-branes is studied semi-classically. The conditions for preserved supersymmetry are derived, leading to a differential equation for the shape of the D3-brane. The solutions of this equation are analyzed. For a D3 brane intersecting the NS5-branes, the angle of approach is known to be restricted to discrete values. Four different ways to obtain this quantization are described. In particular, it is shown that, assuming the D3-brane avoids intersecting a *single* NS5 brane, the above discrete values correspond to the different possible positions of the D3-branes among the NS5-branes.

Keywords: Superstrings and Heterotic Strings, D-branes.

JHEP08(2000)030

Contents

1. Introduction and summary

The dynamics of branes in string theory is an important field of active research, which is hoped to provide clues about the fundamental description of the theory. There are several approaches to this subject, leading to complementary information. Treating the branes as classical manifolds, many results can be obtained using duality and supersymmetry arguments. Alternatively, the branes can be represented by the supergravity background that they induce, providing an additional point of view. A particularly fruitful approach is to represent the NS5-brane by its induced geometry and to consider D-branes in this background. First, the NS5 geometry is relatively simple so, considering the D-branes as classical manifolds, the resulting equations for the shape of the brane are tractable. Second, the Ramond-Ramond fields vanish in this geometry, so the worldsheet formulation of string theory in this background is well understood and can be used. In this formulation, the D-branes are represented by open strings with specified boundary conditions. Finally, the near-horizon region of the NS5-brane geometry corresponds to a *solvable* worldsheet description, enabling the derivation of exact results.

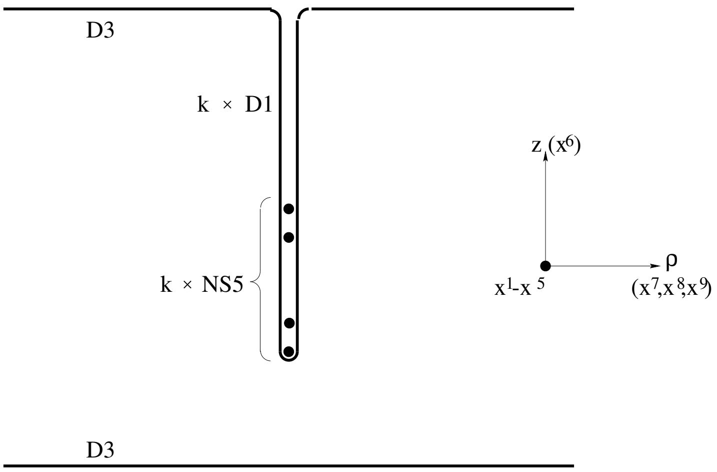

Figure 1: The brane configuration.

In this work we consider the following static brane configuration in type-IIB string theory (see figure 1).[1] orthogonal to the NS5-branes and to $\hat{z}$. This configuration has an SO(3) symmetry and preserves 1/4 of the 32 supercharges in the theory. One can add a D1-brane along the z axis without breaking the above symmetries. Moreover, by moving the D3-brane in the z direction, such a D1-brane is created as the D3-brane crosses an NS5-brane. This is known as the Hanany-Witten effect [1]. In the present treatment, the NS5-branes are represented by the geometry that they induce [2], which is of the form

$$\mathbb{R}^{5,1} \times \mathcal{M}$$

and the D3-brane is considered as a classical 3-manifold D in the 4-dimensional geometry $\mathcal{M}$ transverse to the NS5-branes. Because of the SO(3) symmetry, the shape of this manifold is described by a single function $z(\rho)$. The supersymmetry of this system is analyzed and it is shown (following the approach in [3, 4]) that the preservation of supersymmetry translates to a restriction on the shape of the brane, expressed by a differential equation for $z(\rho)$. Analyzing the solutions of this equation, the Hanany-Witten effect is reproduced: the D3-brane approaches, for $\rho \to \infty$, a flat hyper-surface at $z = z_\infty$, where z_∞ is a continuous parameter; starting with a flat D3-brane at $z = -\infty$ (see figure 1), and moving it "up" (increasing z_∞), the D3-brane is repelled from the NS5-branes and, as a result, a tube is formed; when the D3-brane is high enough, the configuration looks for a distant observer as a 1-brane along the z axis extended between the NS5-branes and a flat D3-brane; by calculating the charge and tension[2] of this 1-brane, it is identified as k coinciding D1-branes.

[1]Using T-duality in directions parallel to the NS5-branes, the D3 brane can be replaced by a D$(3+p)$-brane with p direction parallel to the NS5-branes, without influencing the results obtained here.

[2]The tension is calculated using the Dirac-Born-Infeld action for a D-brane.

The family of configurations described above is part of a larger family, with an additional parameter ψ_0, which characterizes the 2-form field $\mathcal{F}$ on the brane (the above sub-family corresponds to $\psi_0 = \pi$). Classically, this parameter can take any value, however, when quantum considerations are taken into account, one finds that only discrete values are allowed:

$$\psi_0 = N\frac{\pi}{k}\,. \tag{1.1}$$

Of particular interest are configurations in which all the k NS5-branes coincide and the D3-brane intersects (touches) them. This corresponds to the range $0 < \psi_0 < \pi$ and in this case ψ_0 is the angle at which the D3-brane approaches the NS5-branes. The restriction (1.1) implies in this case that only $k-1$ values are allowed for this angle, corresponding to $N = 1, \ldots, k-1$.

Remarkably, there are four independent ways to derive the integrality condition (1.1):

1. Demanding that the coupling of the fundamental string to the 2-form field $\mathcal{F}$ on the D-brane is well defined;

2. Demanding that the number of D1-branes ending on the D3-brane is integral;

3. Assuming that a D3-brane does not intersect a single NS5-brane;

4. Using the worldsheet description in the near-horizon region, where exact quantum analysis can be performed.

We describe the first approach in general (following [5]) and show that, in special situations, it translates to an integrality condition

$$\frac{1}{2\pi}\int_{C_2} F \in \mathbb{Z}\,, \qquad \forall C_2 \in H_2(D,\mathbb{Z}) \tag{1.2}$$

on the gauge field strength F on the D-brane. This condition was proposed recently in [6] (and discussed further in [7]–[12]) and here this proposal is confirmed and clarified. Applied to the present case, it leads to the condition (1.1). In the second approach, a D3-brane with $\psi_0 > 0$ is considered. For z_∞ large enough, such a configuration includes a tube connected to the assympoticly flat part of the D3-brane (as illustrated, for $\psi_0 = \pi$, in figure1). For $z_\infty \to \infty$, the tube shrinks to zero width and it is shown to have a charge and tension[2] of $N = k\psi_0/\pi$ D1-branes. Integrality of N leads, again, to the condition (1.1).

In both these approaches, the integrality condition (1.1) is derived from a consistency requirement and leaves the real origin of the restriction obscure. The third approach provides a more natural starting point. The NS5-branes are distributed along the z axis, and it is shown that an integer N in eq. (1.1) corresponds to a D3-brane passing between the Nth and the $(N+1)$th NS5-brane, while for $\hat{N}-1 < N < \hat{N}$ (with $\hat{N} = 1, \ldots, k$) the D3-brane intersects the $\hat{N}$th NS5-brane. Thus, the restric-

JHEP08(2000)030

tion (1.1) simply means that the D3 and NS5-branes avoid intersecting each-other and they do intersect only if this is unavoidable, as is the case when a D3-brane is trapped between coinciding NS5-branes.

Finally, in the forth approach, one restricts attention to the near-horizon region of coinciding NS5-branes and considers the formulation of the string dynamics in this background in terms of the corresponding 2D conformal field theory. The D-branes studied so far in this approach are described in spherical coordinates by $\psi = \psi_0$ (i.e., ψ independent of the radial coordinate r). Note that this is only a subset of the configurations found in the semi-classical approach described above: ψ_0 is restricted to the range $(0, \pi)$ and, for each such ψ_0, there is a single value of z_∞. Imposing the symmetry requirements, one obtains (following the approach in [13]) $k-1$ types of branes. A calculation of the expectation values of bulk fields [14] provides evidence that these branes correspond to the values in eq.(1.1) with $N = 1, \ldots, k-1$. It is worth noting that there are no additional branes corresponding to $\psi_0 = 0, \pi$, i.e., to D1-branes. Instead, D1-branes are seen in the near-horizon region as cylindrical D3-brane. For a single D1 brane, the radius of this cylinder is comparable or smaller to the string scale, so it indeed should be seen as a 1-brane. However, for a large number of coinciding D1-branes, this can be a wide and nearly flat D3-brane. This "condensation" of D1-branes to form a D3-brane is the dielectric effect observed in [15] (see also [16]).

The structure of this work is as follows. In the next section, the supersymmetry of the system is considered and a differential equation for the shape of the D3-brane is derived. In section 3, the solutions of this equation are analyzed and in the last section, the quantization of the parameter ψ_0 is discussed.

2. Preserved supersymmetry

Type-II string theory in flat space-time, has supersymmetry parametrized by two Weyl-Majorana spinors ξ_L, ξ_R (originating, respectively, from the left-moving and right-moving sectors). In the type-IIB theory, both spinors have the same chirality and can be combined to a single complex Weyl spinor

$$\xi = \xi_L + i\xi_R \,. \tag{2.1}$$

In the conventions used in this work, the chirality condition is

$$\overline{\Gamma}\xi = -\xi \,, \tag{2.2}$$

where

$$\{\Gamma_A, \Gamma_B\} = 2\eta_{AB} \,, \qquad \eta = \mathrm{diag}\{-, +, \ldots, +\} \,, \tag{2.3}$$

$$\Gamma_A^\dagger = \Gamma_0 \Gamma_A \Gamma_0 \,,$$

$$\Gamma_{A_1 \ldots A_r} = \Gamma_{[A_1} \ldots \Gamma_{A_r]} \,, \qquad \overline{\Gamma} = \Gamma_{0 \ldots 9} \,. \tag{2.4}$$

The Majorana condition is

$$(\xi_{L,R})_c = \xi_{L,R}\,, \tag{2.5}$$

where $\xi \to \xi_c$ is "charge-conjugation":

$$\xi_c = D\xi^*\,, \qquad D^{-1}\Gamma_A D = -\Gamma_A^*\,, \qquad D^* D = 1\,. \tag{2.6}$$

(so, for ξ in eq. (2.1), $\xi_c = \xi_L - i\xi_R$).

In a background including branes and/or bulk fields, at least some of this supersymmetry is broken. In this section we identify the supersymmetry that remains unbroken in the background considered in this work. First we consider the supersymmetry preserved by the NS5 background and then, the further influence of D3-branes is analyzed.

2.1 The NS5 geometry

The geometry (metric ds^2 in the string frame, dilaton Φ and NS 3-form field strength H) induced by k coinciding flat NS5-branes is [2]

$$ds^2 = dx^2 + h dy^2\,, \tag{2.7}$$

$$e^{2\Phi} = g_s^2 h\,, \tag{2.8}$$

$$H = -*_4\, dh \qquad (H_{klm} = -\epsilon_{klmn}\partial_n h), \tag{2.9}$$

where $x^\mu = (x^0, x^1, x^2, x^3, x^4, x^5)$ parametrize the directions parallel to the NS5-branes, $y^m = (y^6, y^7, y^8, y^9)$ correspond to the transverse directions and g_s is the string coupling far from the brane. At this stage we consider coinciding branes, for which h is the following harmonic function on the transverse space

$$h = 1 + \frac{kl_s^2}{r^2}\,, \qquad r = |y| = \sqrt{k} l_s e^\phi\,, \tag{2.10}$$

where l_s is the string coupling. In this case, eqs. (2.7), (2.9) can be rewritten as

$$ds^2 = dx^2 + hr^2(d\phi^2 + d\Omega_3^2)\,, \tag{2.11}$$

$$H = 2kl_s^2\omega_3\,, \tag{2.12}$$

where $d\Omega_3^2$ and ω_3 are the metric and volume form on the unit 3-sphere S^3_{6789}. Far from the branes (for $r \gg \sqrt{k} l_s$), $h \approx 1$ and the geometry is trivial: Minkowski space with a constant Φ and a vanishing H. Close to the brane (for $r \ll \sqrt{k} l_s$), $hr^2 \approx kl_s^2$ and the geometry is that of a throat

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times S^3_{6789}\,, \tag{2.13}$$

with an H flux through S^3_{6789} and a dilaton Φ depending linearly on ϕ. In most of this work, this geometry will be considered semi-classically. For this to be a justified approximation, one must assume:

JHEP08(2000)030

- Small string coupling (loop expansion parameter: $e^{\Phi} \ll 1$:

  this is always violated for r close enough to the NS5-branes; however, for g_s sufficiently small, the strongly-coupled region can be pushed arbitrarily deep into the throat.

- Small curvature:

  this is always true outside the throat; the throat is also weakly-curved whenever $k \gg 1$.

We look for the supersymmetry transformations that preserve this geometry, i.e., for which the variation of all the fields vanishes. Since all the fermionic fields vanish, the only non trivial conditions come from the variation of the fermionic fields – the complex Weyl spinor λ and the (complex) gravitino ψ_M. The resulting equations are [17]:

$$0 = \quad \delta\lambda \quad = i\left(P_M\Gamma^M\xi_c - \frac{1}{24}G_{NKL}\Gamma^{NKL}\xi\right), \tag{2.14}$$

$$0 = \delta\psi_M = D_M\xi + \frac{1}{96}\left(g_{MM'}G_{NKL}\Gamma^{M'NKL} - 9G_{MKL}\Gamma^{KL}\right)\xi_c\,, \tag{2.15}$$

where

$$P = \frac{1}{2}d\Phi\,, \qquad G = \sqrt{g_s}e^{-\Phi/2}H \tag{2.16}$$

and g_{MN} is the metric in the *Einstein* frame, and is related to the metric g^{st}_{MN} in eq. (2.7) by

$$g_{MN} = \sqrt{g_s}e^{-\Phi/2}g^{st}_{MN}\,.$$

The covariant derivative of a spinor is

$$D_M\xi = \left(\partial_M + \frac{1}{4}\Omega_{MKL}\Gamma^{KL}\right)\xi\,, \tag{2.17}$$

$$\Omega_{MKL} = \hat{\Omega}_{KLM} + \hat{\Omega}_{KML} + \hat{\Omega}_{MLK}\,, \qquad \hat{\Omega}_{MKL} = e^A_L\partial_{[M}e_{K]A}\,,$$

where $\{e^A = e^A_M dx^M\}$ is an orthonormal frame

$$g_{MN} = \eta_{AB}e^A_M e^B_N$$

and the matrices Γ^M are related to those in eq. (2.3) by

$$\Gamma_A = \eta_{AB}e^B_M\Gamma^M\,.$$

For the geometry (2.7)-(2.9), eqs. (2.14), (2.15) simplify to

$$0 = \delta\lambda \sim \xi_c + \Gamma_{6789}\xi\,, \qquad 0 = \delta\psi_\mu = \partial_\mu\xi\,, \qquad 0 = \delta\psi_m \sim \partial_m(h^{1/16}\xi)\,, \tag{2.18}$$

and the general solution is

$$\xi = \xi_0 h^{\frac{1}{16}}\,, \qquad \Gamma_{6789}\xi = -\xi_c\,, \tag{2.19}$$

where ξ_0 is a constant spinor.

2.2 D3-branes

Next we consider D3-branes in the above background. The supersymmetry preserved by such a brane configuration depends on the charges it carries. D-branes carry RR charges. A D3-brane carries a charge Q_3, coupling to the RR 4-form $C_{[4]}$ and, in some situations (as the present one), also a charge Q_1 ("D1 charge"), coupling to the RR 2-form $C_{[2]}$. This is expressed by the low-energy effective action of the D3-brane (see for example [18])

$$S = g_s T_3 \left[-\int d^4\zeta e^{-\Phi} \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + i \int (C_{[4]} + \mathcal{F} \wedge C_{[2]}) \right] , \qquad (2.20)$$

where

$$T_p = \frac{1}{g_s l_s (2\pi l_s)^p} \qquad (2.21)$$

is the tension of a Dp-brane in a trivial background, $G_{\mu\nu}$ is the bulk metric (pulled-back to the worldvolume of the brane) and $\mathcal{F}$ is a 2-form living on the brane such that $d\mathcal{F} = H$ is the bulk NS 3-form (also pulled back to the worldvolume of the brane). The 2-form $\mathcal{F}$ is conventionally decomposed as

$$\mathcal{F} = B + 2\pi l_s^2 F \, , \qquad (2.22)$$

where B is the bulk Kalb-Ramond field (with $dB = H$) and F is a U(1) gauge field strength on the brane (with $dF = 0$)[3].

To find the preserved supersymmetry in the presence of such charges, we consider first the simpler situation of D-branes in flat space and then return to the case of interest.

2.2.1 D-branes in flat space

Consider type-IIB string theory compactified on a (rectangular) 3-torus T^3_{123} with static N_3 D3-branes and N_1 D1 branes wrapped on the torus, the D1-branes extended in the x^1 direction and delocalized evenly in the other directions of the torus (so that translational symmetry is preserved). The supersymmetry algebra in this situation is (see for example [18]

$$\{\hat{Q}, \hat{Q}^\dagger\} = -2 \begin{pmatrix} M & (M_3\beta_{123} + M_1\beta_1)\Gamma_0 \\ \Gamma_0^\dagger (M_3\beta_{123} + M_1\beta_1)^\dagger & M \end{pmatrix} , \qquad \beta_A = \Gamma_A \overline{\Gamma} \, , \qquad (2.23)$$

where $\hat{Q} = (Q_L, Q_R)$ are the supercharges, M is the total mass of the state and

$$M_p = N_p T_p V_p$$

[3] Note that the action (2.20) is well defined only when H is exact in some neighborhood of the D brane. Note also that the decomposition (2.22) is not unique. The freedom to choose different such decompositions is the gauge freedom associated with the 2-form potential B.

is the mass of N_p coinciding Dp-branes of volume V_p. A preserved supersymmetry in a given state corresponds to a spinor $\hat{\xi} = (\xi_L, \xi_R)$ (ξ_L, ξ_R being two Weyl-Majorana spinors with $\overline{\Gamma} = -1$) for which $\hat{Q}^\dagger \hat{\xi}$ vanishes in that state and this implies that $\hat{\xi}$ is an eigenvector of the matrix on the r.h.s. of eq. (2.23) with a vanishing eigenvalue. The resulting "BPS condition" is

$$M\xi = i(M_3 \Gamma_{0123}\xi + M_1 \Gamma_{01}\xi_c)\,, \qquad \xi = \xi_L + i\xi_R\,. \tag{2.24}$$

"Squaring" eq. (2.24), one obtains that the mass of such a BPS configuration must be

$$M = \sqrt{M_3^2 + M_1^2}\,. \tag{2.25}$$

Note that this is less then the sum of the masses of the constituents, so this is a real (non-threshold) bound state of the D3 and D1-branes[4].

The condition (2.24) is an integrated version of a corresponding local condition. Because of the homogeneity of the present configuration, this local condition can be identified simply by dividing by the volume of the torus. The result is

$$T\xi = iT_3(N_3 \Gamma_{0123}\xi + n_1 \Gamma_{01}\xi_c)\,, \qquad T = T_3\sqrt{N_3^2 + n_1^2}\,, \tag{2.26}$$

where

$$\frac{n_1}{(2\pi l_s)^2} = \frac{N_1}{V_{23}} \tag{2.27}$$

is the "number density" of the D1-branes.

Finally, consider a configuration with a single D3-brane, in which the D1 branes are replaced by a 2-form field $\mathcal{F}$ on the worldvolume of the D3 brane, with the same charges and preserved symmetries as above. Translational symmetry implies that $\mathcal{F}$ is constant. To identify its value, we consider the resulting coupling to the RR 2-form $C_{[2]}$. From the D3-brane action (2.20) one obtains

$$ig_s T_3 \int_{D3} \mathcal{F} \wedge C_{[2]}$$

and this should be equated with the coupling of N_1 D1-branes in the x^1 direction

$$iN_1 g_s T_1 \int_{D1} C_{[2]}\,.$$

This implies

$$\mathcal{F} = n_1 \omega_{23}\,, \tag{2.28}$$

where ω_{23} is the volume form on the 2-torus T^2_{23} transverse to the D1-branes and n_1 is related to the density of the D1 branes by eq. (2.27).

We are ready now to return to the D3-branes in the NS5 background

[4] Performing T-duality in the x^2 direction, one obtains two types of D2-branes, extended in the (13) and (12) directions respectively. In this situation, the BPS bound state is naturally identified as a configuration with parallel D2-branes with orientation $dx^1 \wedge (M_3 dx^3 + M_1 dx^2)$.

2.2.2 D3-branes in the NS5-brane geometry

We consider a configuration that is static, preserves the $SO(3)_{789}$ symmetry and in which the D3-brane is orthogonal to the NS5-branes (i.e. its worldvolume is restricted to fixed x^i, $i = 1, \ldots, 5$). In this context, it is useful to use also cylindrical coordinates $(z, \rho, \theta, \varphi)$

$$(y^6, y^7, y^8, y^9) = (z, \rho(\cos\theta, \sin\theta(\cos\varphi, \sin\varphi))) \tag{2.29}$$

($\theta \in [0, \pi]$; $\varphi \sim \varphi + 2\pi$) and spherical coordinates, replacing (z, ρ) by $\phi \in \mathbb{R}$, $\psi \in [0, \pi]$:

$$(z, \rho) = r(\cos\psi, \sin\psi)\,, \qquad r = \sqrt{k}l_s e^{\phi}\,. \tag{2.30}$$

In these coordinates (see eqs. (2.11), (2.12)),

$$\begin{aligned}
ds^2 &= dx^2 + h(dz^2 + d\rho^2 + \rho^2 d\Omega_2^2) \\
&= dx^2 + hr^2(d\phi^2 + d\psi^2 + \sin^2\psi d\Omega_2^2)\,, \\
H &= 2kl_s^2 \frac{\rho^2}{(z^2+\rho^2)^2}(zd\rho - \rho dz) \wedge \omega_2 \\
&= 2kl_s^2 \sin^2\psi d\psi \wedge \omega_2\,,
\end{aligned} \tag{2.31}$$

where $d\Omega_2^2$ and ω_2 are the metric and volume form on the unit 2-sphere S^2_{789}

$$d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2\,, \qquad \omega_2 = \sin\theta d\theta \wedge d\varphi\,.$$

In the configuration of the above type, the D3-brane is wrapped on S^2_{789} and is extended in an additional direction

$$t \propto \sin\chi dz + \cos\chi d\rho\,,$$

where χ is independent of the S^2_{789} coordinates. Because of the rotational symmetry, it is enough to consider $y^8 = y^9 = 0$, $y^7 \geq 0$ and there $t \propto \sin\chi dy^6 + \cos\chi dy^7$, so the non-vanishing components of t are

$$(t_6, t_7) = (\sin\chi, \cos\chi)\,. \tag{2.32}$$

We assume that the 2-form $\mathcal{F}$ on the D-brane is proportional to ω_2 (and, therefore, $SO(3)_{789}$-symmetric)

$$\mathcal{F} = kl_s^2 f(\psi)\omega_2\,. \tag{2.33}$$

Integrating $d\mathcal{F} = H$, one obtains[5]

$$f(\psi) = \psi - \psi_0 - \frac{1}{2}\sin 2\psi\,. \tag{2.34}$$

[5] Identifying $B = [B_0 + kl_s^2(\psi - \frac{1}{2}\sin 2\psi)]\omega_2$ (see eq. (2.22)), this corresponds to the case of a purely-magnetic field-strength on the D-brane: $F = F_0\omega_2$ ($B_0 + 2\pi l_s^2 F_0 = -kl_s^2\psi_0$).

The above configuration is very similar to the configuration in flat space considered before: a D3-brane wrapped on a homogeneous compact 2D surface S (a 2-sphere in the present case), with an $\mathcal{F}$ field proportional to the volume form on S. The BPS condition is, therefore, eq. (2.26) translated to the present notation:

$$\sqrt{1+n_1^2}\,\xi = i(t_a\Gamma_{0a89}\xi + n_1 t_a \Gamma_{0a}\xi_c)\,, \tag{2.35}$$

where t_a is defined in eq. (2.32). To identify n_1, one notes that the radius of the 2-sphere, on which the D3 brane is wrapped, is

$$R_2 = \rho\sqrt{h} = l_s\sqrt{k(1+e^{2\phi})}\sin\psi\,, \tag{2.36}$$

so the volume form is $R_2^2\omega_2$ and, comparing eq. (2.33) to (2.28), one obtains

$$n_1 = \mathcal{F}_{\hat\theta\hat\varphi} = \frac{f}{g}\,, \qquad g = \frac{\rho^2 h}{k l_s^2} = (1+e^{2\phi})\sin^2\psi\,. \tag{2.37}$$

We will consider a D3-brane which, far from the NS5-brane, is flat, extended in the (789) directions and has a vanishing n_1. The restriction on preserved supersymmetry imposed by this part of the brane is

$$\xi = i\Gamma_{0789}\xi\,. \tag{2.38}$$

Combining this with the restriction imposed by the NS5-brane (eq. (2.19)), one obtains

$$\xi = -i\Gamma_{06}\xi_c \tag{2.39}$$

(which is the restriction imposed by a D1 brane extended in x^6). Substituting eqs. (2.38), (2.39) in the BPS condition (2.35), one obtains

$$\sqrt{1+n_1^2}\,\xi = t_a\Gamma_a(\Gamma_7 - n_1\Gamma_6)\xi = (1+n_1\Gamma_{67})e^{\chi\Gamma_{67}}\xi$$

(where in the second equality one uses $\Gamma_{67}^2 = -1$ and the explicit expression (2.32) for t_a). Γ_{67} anti-commutes with Γ_{0789}, therefore, this equation can be compatible with eq. (2.38) only if it is an identity, which can be written as

$$1 + n_1\Gamma_{67} = \sqrt{1+n_1^2}\,e^{-\chi\Gamma_{67}}\,. \tag{2.40}$$

This is equivalent to $\arg(1-in_1) = \chi$ and, therefore, also to

$$\tan\chi = -n_1\,.$$

Finally, using the definition (2.32) of χ and the expression (2.37) for n_1, the condition for preserved supersymmetry translates to the following differential equation

$$\frac{dz}{d\rho} = -\frac{f}{g} \tag{2.41}$$

and, in spherical coordinates[6],

$$\frac{d\phi}{d\psi} = \frac{\mathcal{A}_0}{\mathcal{A}_1}\,, \tag{2.42}$$

$$\begin{aligned}
\mathcal{A}_0 &= g\sin\psi - f\cos\psi = \sin\psi - (\psi-\psi_0)\cos\psi + e^{2\phi}\sin^3\psi\,,\\
\mathcal{A}_1 &= g\cos\psi + f\sin\psi = (\psi-\psi_0)\sin\psi + e^{2\phi}\sin^2\psi\cos\psi\,.
\end{aligned}$$

It is constructive to compare this equation to the corresponding BPS equation in a trivial background. It can be reproduced from eq. (2.41) by considering a very large distance r from the NS5-branes. This means that $h \approx 1$ and f (which is a function of z/ρ) is approximately constant, representing a closed $\mathcal{F}$: $\mathcal{F} = \mathcal{F}_0\omega_2$. The resulting equation is [3]

$$\frac{dz}{d\rho} = -\frac{\mathcal{F}_0}{\rho^2}\,.$$

Note that, for a D3-brane that extends over regions with different z/ρ, the configuration of the brane may differ significantly from that in a trivial background, even if the brane is everywhere far from the NS5 branes.

We end this section with a comment on antibranes. NS5-branes and antibrane differ in the sign of the 3-form field strength H (eq. (2.9)). D3-branes and antibranes differ in the sign of their RR charges (the second term in eq. (2.20)). In both types of branes, this leads to a sign change in the equation for the preserved supersymmetry (eqs. (2.19) and (2.38) respectively), but there is *no change* in the differential equation (2.41), so in all cases the D3 brane has the same shape.

3. BPS D3-branes

In this section we analyze the solutions of the BPS equation (2.41)

$$\frac{dz}{d\rho} = -\frac{kl_s^2 f}{\rho^2 h} = -\frac{1}{1+e^{2\phi}}\cdot\frac{f(\psi)}{\sin^2\psi}\,. \tag{3.1}$$

Such an analysis was performed, for a D5-brane in the geometry induced by D3-branes, in [19] and the results are qualitatively the same. This should be expected, since the configurations are related by duality.

3.1 The shape of the brane

The solutions of eq. (2.41) have the following properties:

- The BPS equation is invariant under $\psi \to \pi - \psi$ ($z \to -z$) and $\psi_0 \to \pi - \psi_0$, so it is enough to consider $\psi_0 \geq \frac{\pi}{2}$.

[6]Using $r^2 d\phi = zdz + \rho d\rho$, $r^2 d\psi = zd\rho - \rho dz$.

- For $\rho \neq 0$, $dz/d\rho$ is finite, so the D3 profile is described by a smooth function $z(\rho)$ in the interval $0 < \rho < \infty$. The sign of its derivative is governed by $f(\psi)$.

- For $\rho \to \infty$: $dz/d\rho \to 0$, so $\psi \to \pi/2$. For $\psi_0 \neq \pi/2$ this leads to

$$z \approx z_\infty + \left(\frac{\pi}{2} - \psi_0\right) \frac{kl_s^2}{\rho} \tag{3.2}$$

  (with z_∞ being an integration constant).

- The behavior at $\rho \to 0$ depends on the value of ψ_0. It can be shown that in this limit, $z \to 0$ iff $0 < \psi_0 < \pi$. Considering first the case of $z \not\to 0$, for $\psi_0 = \pi$ (implying $f \to 0$) the solution is

$$z \approx -r_0 + \frac{kl_s^2}{3r_0(kl_s^2 + r_0^2)}\rho^2 \,,$$

  while for $\psi_0 > \pi$, it is

$$z \approx -\frac{(\psi_0 - \pi)kl_s^2}{\rho} + z_0$$

  (with $r_0 > 0$ and z_0 being integration constants).

- When both z and ρ are small (compared to $\sqrt{k}l_s$), the last term in $\mathcal{A}_0, \mathcal{A}_1$ (see eq. (2.42)) is negligible and the BPS condition becomes

$$\frac{d\phi}{d\psi} = \frac{1}{\psi - \psi_0} - \cot\psi \,. \tag{3.3}$$

  It is solved by

$$r = r_0 \frac{|\psi - \psi_0|}{\sin\psi} \tag{3.4}$$

  (where r_0 is an integration constant) and by $\psi = \psi_0$ (which is the $r_0 \to \infty$ limit of the solution (3.4)).

The resulting configurations are illustrated in figures 2, 3 and 4. They are parametrized by ψ_0, which represents the value of $\mathcal{F}$ and, for each such value, there is an additional continuous parameter that can be identified with z_∞ in eq. (3.2). Far from the NS5-branes, the configuration is the trivial one[7]: a flat D3-brane (at $z = z_\infty$) with a vanishing $\mathcal{F}$. The shape of the brane depends continuously on z_∞ (i.e., the topology does not change), while in the dependence on ψ_0, there is a qualitative change at $\psi_0 = \pi$ (and similarly at $\psi_0 = 0$):

- $\psi_0 = \pi$ (figure2):

  For z_∞ negative and large enough, the whole brane is nearly flat; as z_∞ is increased, the NS5-branes repel the D3-brane and, as a result, a (finite) tube is formed.

[7]Recall that this was *a choice* (see below eq. (2.37)) that was used to derive the BPS equation (2.41).

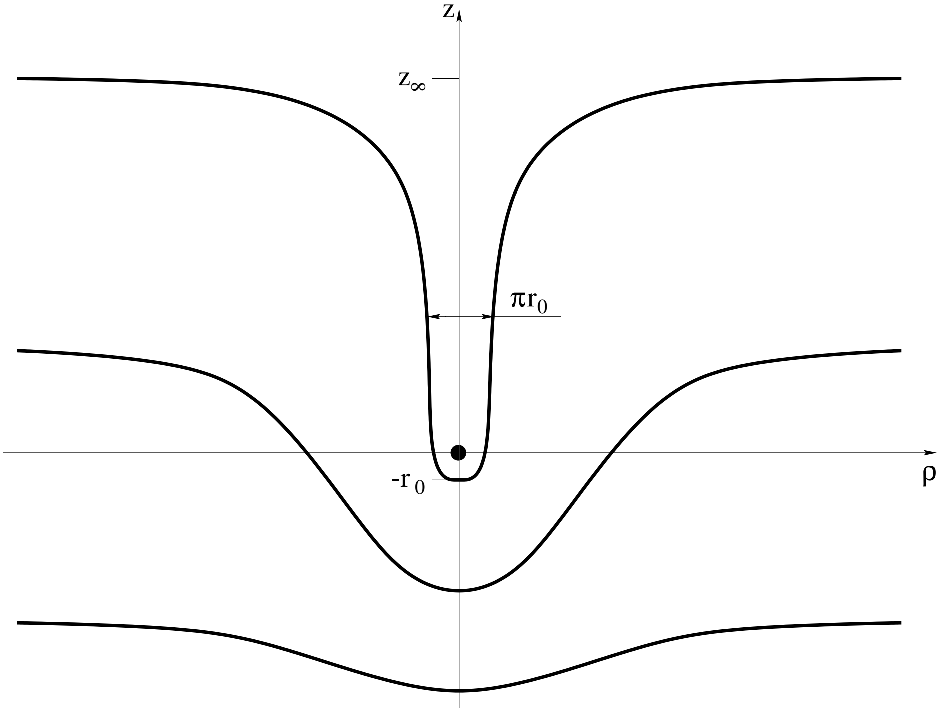

Figure 2: D3-brane profiles with $\psi_0 = \pi$.

- $\psi_0 > \pi$ (figure 3):

  For any z_∞, the D3-brane avoids the $\rho = 0$ line[8] and an infinite tube is formed, extending to $z \to -\infty$.

- $0 < \psi_0 < \pi$ (figure 4):

  These configurations also include an infinite tube, but this time it extends to $\phi \to -\infty$ in the throat, reaching the NS5-branes. In this case, ψ_0 has a clear geometrical meaning: deep in the throat, the D3-brane approaches a cylinder with $\psi = \psi_0$ (independently of z_∞), so this is the direction from which the D3-brane approaches the NS5-branes.

For any ψ_0, by changing z_∞ in the appropriate direction, the tube can be made arbitrarily narrow. When the radius R_2 of the tube (eq. (2.36)) becomes comparable to l_s (or smaller), the semi-classical description used here breaks down and the tube is more appropriately interpreted as a 1-brane. The resulting brane configuration, as seen at large scales, is composed of a flat D3-brane and a 1-brane ending on the D3-brane, both branes being orthogonal to each-other (and to the NS5-branes). In the configuration in figure 3, the 1-brane is semi-infinite, while in the other two cases,

[8] In fact, the D3-brane avoids the $\rho = 0$ for any $\psi_0 \neq 0, \pi$. This means that the brane avoids the singularities of $\mathcal{F}$. These singularities can be deduced from the components of $\mathcal{F}$ in an orthonormal frame. The non-vanishing component $\mathcal{F}_{\hat{\theta}\hat{\varphi}}$ is given in eq. (2.37), so $\mathcal{F}$ is singular at $\psi = 0$ (for $\psi_0 \neq 0$) and at $\psi = \pi$ (for $\psi_0 \neq \pi$).

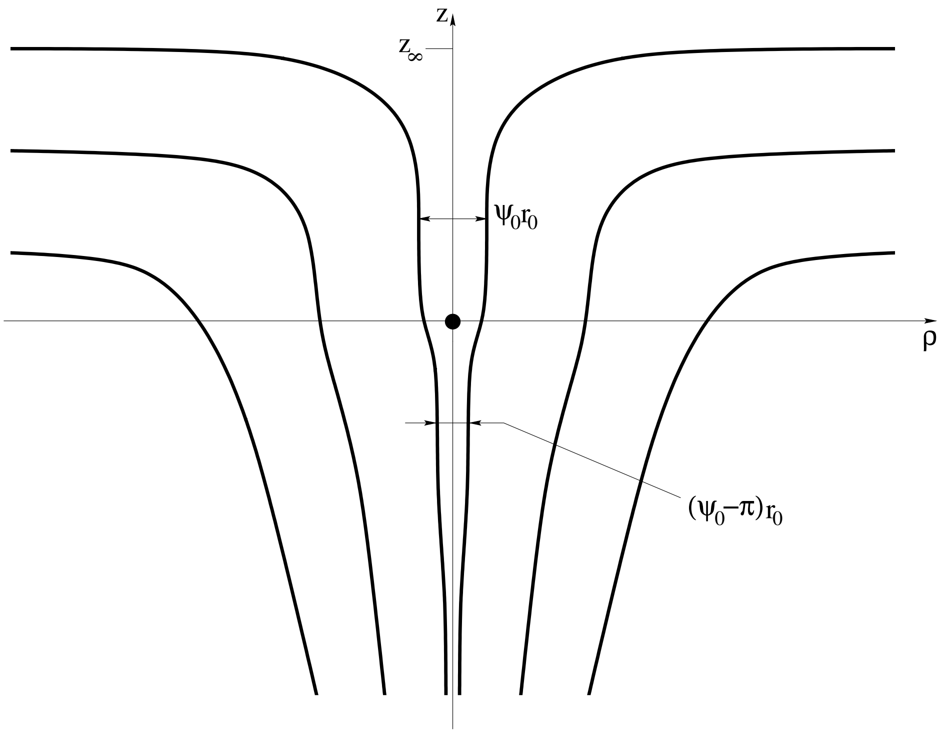

Figure 3: D3-brane profiles with $\psi_0 > \pi$.

the 1-brane has a finite extension, its lower end being at the NS5-branes. The nature of this 1-brane will be discussed in the next section.

3.2 The energy of the brane

The low-energy effective action for the D3-brane is given in eq. (2.20). For a static brane, it takes the form $S = -E\Delta x^0$, where E is the energy of the brane (measured w.r.t. the time coordinate x^0). In the present configuration, the energy is a functional of $z(\rho)$

$$E = \frac{k}{\pi} T_1 \int d\rho \sqrt{z'^2 + 1}\, \mathcal{A}\,, \qquad \mathcal{A}^2 = g^2 + f^2 \tag{3.5}$$

(where g, f are given in eqs. (2.37), (2.34) respectively and the $z' = dz/d\rho$) and the equation obtained from extremizing it is

$$\frac{d}{d\rho}\left[\frac{z'\mathcal{A}}{\sqrt{z'^2+1}}\right] = \sqrt{z'^2+1}\frac{\partial\mathcal{A}(z,\rho)}{\partial z}\,. \tag{3.6}$$

One can show that any solution of the BPS condition (2.41) also exrtemizes the energy (i.e. solves eq. (3.6))[9].

[9]One way to do this is as follows: using

$$\mathcal{A}^2 = g^2 + f^2\,, \qquad z' = -\frac{f}{g}\,,$$

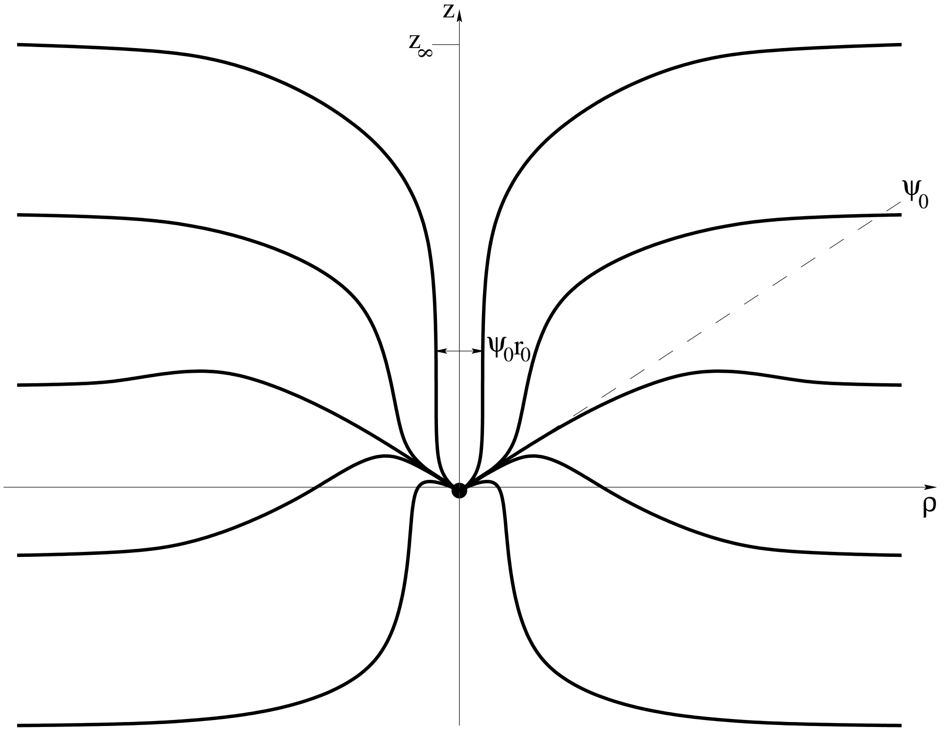

Figure 4: D3-brane profiles with $0 < \psi_0 < \frac{\pi}{2}$.

4. Quantization

So far, the discussion was purely classical. The quantum theory induces quantization restrictions on the parameters appearing in the classical description. These can be derived in various ways and lead to identical results.

4.1 Consistency of the fundamental string interactions

The quantization restrictions can be derived from the coupling of the fields H and $\mathcal{F}$ to the fundamental string (this was analyzed in [5] and considered further in [7, 9, 12], [20]–[23]. Consider such a string, propagating in a spacetime $\mathcal{M}$. Its worldsheet $\Sigma \subset \mathcal{M}$ may have a boundary $\partial\Sigma$ which, however, is necessarily confined to the worldvolume $D \subset \mathcal{M}$ of a D-brane. Assuming that there exists a three-dimensional manifold $\hat{\Sigma} \in \mathcal{M}$ such that[10] $\partial\hat{\Sigma} = \Sigma + \Sigma_D$, $\Sigma_D \subset D$, the coupling of the string to

eq. (3.6) simplifies to

$$\frac{\partial f}{\partial \rho} = -\frac{\partial g}{\partial z},$$

where f, g are considered as function of z, ρ and one can verify that f, g given in eqs. (2.33), (2.34) indeed satisfy this last equation.

[10]The existence of such $\hat{\Sigma}$ for any Σ is expressed mathematically as the triviality of the relative homology group $H_2(\mathcal{M}/D, \mathbb{Z})$. In the present case, $H_2(\mathcal{M}, \mathbb{Z}) = H_1(D, \mathbb{Z}) = 0$, which indeed implies $H_2(\mathcal{M}/D, \mathbb{Z}) = 0$. The more general case $H_1(D, \mathbb{Z}) \neq 0$ was considered in [5].

the fields H and $\mathcal{F}$ is

$$S_{\rm Int} = \frac{1}{2\pi l_s^2}\left(\int_{\hat\Sigma} H - \int_{\Sigma_D} \mathcal{F}\right). \tag{4.1}$$

Note that in topologicly-trivial situations, when

$$H = dB\,, \qquad F \equiv \frac{\mathcal{F} - B}{2\pi l_s^2} = dA\,, \tag{4.2}$$

eq. (4.1) reduces to the following familiar form:

$$S_{\rm Int} = \frac{1}{2\pi l_s^2}\int_{\Sigma} B + \int_{\partial\Sigma} A\,.$$

The phase $e^{iS_{\rm Int}}$ (appearing in the path integral) must be independent of the choice of $\hat\Sigma$ and this translates to

$$\frac{1}{2\pi l_s^2}\left(\int_{C_3} H - \int_{C_2} \mathcal{F}\right) \in 2\pi\mathbb{Z}\,, \tag{4.3}$$

for any $C_3 \subset \mathcal{M}$ with $\partial C_3 = C_2 \subset D$. In particular, for a closed C_3 (which always the case for a closed string) the condition is

$$\frac{1}{(2\pi l_s)^2}\int_{C_3} H \in \mathbb{Z}\,, \tag{4.4}$$

which is an integrality condition on the cohomology class of H. For a given H satisfying the integrality constraint (4.4), the further condition (4.3) with an open C_3 is a condition on the worldvolume D of the D-brane and the field $\mathcal{F}$ on it. Note that C_3 and C_3' with the same boundary C_2 lead to the same condition (because of the integrality condition (4.4)) so, for a given $C_2 \subset D$, one can choose a convenient C_3. When the Kalb-Ramond field B is sufficiently regular, so that C_3 can be chosen to avoid the singular locus of B (which means that H is exact in C_3: $H = dB$), condition (4.3) simplifies (using the decomposition (2.22)) to

$$\frac{1}{2\pi}\int_{C_2} F \in \mathbb{Z}\,. \tag{4.5}$$

This form for the integrality condition was proposed recently in [6] (see also [7, 9, 12]), in the specific case of $\mathcal{M} = S^3$, and here this proposal is confirmed. As pointed out in [6], one should question the meaning of an integrality condition for a quantity like F, that is not gauge invariant (see footnote3). This issue is clarified in the present derivation: to obtain the above condition on F, a (regular) gauge choice of B is necessary.

We now apply the above considerations to the NS5 background. The topology is $\mathbb{R}^7 \times S^3$. The first factor is trivial homologicly and $H, \mathcal{F}$ (in eqs. (2.12), (2.33)), do not depend on it, so one can replace C_3 by its projection on the second factor, which is a subset of S^3. The only closed C_3 is S^3_{6789} itself, so the integrality constraint (4.4) is

$$\frac{1}{(2\pi l_s)^2}\int_{S^3} H \in \mathbb{Z}\,. \tag{4.6}$$

Using eq. (2.12) and $\int \omega_3 = 2\pi^2$, this translates to $k \in \mathbb{Z}$. As to an open C_3, we choose for the bulk Kalb-Ramond field

$$B = kl_s^2(\psi - \frac{1}{2}\sin 2\psi)\omega_2\,, \tag{4.7}$$

which is singular only at $\psi = \pi$ (see footnote8). For C_2 avoiding[11] $\psi = \pi$, one can choose C_3 that also avoids the singular point and use the integrality condition (4.5). Using the explicit expressions for B and $\mathcal{F}$ (eqs. (4.7), (2.33), (2.34)), one obtains

$$F = -kl_s^2\psi_0\omega_2\,, \tag{4.8}$$

which is independent of ψ, so one can project C_2 on S^2_{789}, obtaining an integral multiple of S^2_{789}. The integrality condition (4.5) is, therefore

$$\frac{1}{2\pi}\int_{S^2} F \in \mathbb{Z}\,,$$

which leads to

$$\psi_0 = N\frac{\pi}{k}\,, \qquad N \in \mathbb{Z}\,. \tag{4.9}$$

Note that, although the derivation used a choice of gauge, the result (4.9) has a gauge invariant meaning, since ψ_0 determines the field $\mathcal{F}$ (see eq. (2.34)). Note also that, while the condition (4.5) is a condition on F, the dynamics (i.e., the effective action (2.20) turned it to a restriction on the possible configurations of the D3-brane (as described in the previous section). In particular, for the configuration $\psi = \psi_0$ in the throat region ($r \ll \sqrt{k}l_s$; see the comment below eq. (3.4)), there are only $k-1$ possible 2-spheres on which the D3-brane can be wrapped, corresponding to $N = 1, \ldots, k-1$ (this was first derived in [20]).

4.2 The integrality of the number of branes

Although the integrality of the parameter k was obtained in the previous subsection using a consistency argument, this integrality is obviously also a consequence of the identification of k as the number of NS5-branes. Likewise, as will be shown in this subsection, the integrality condition (4.9) on ψ_0 may be derived from the integrality of the number of D1-branes that can end on the D3-brane.

As described in the previous section, for $\psi_0 > 0$ and z_∞ large enough, the configuration includes a long and thin tube. When the radius R_2 (eq. (2.36)) of this

[11] C_2 passing through $\psi = \pi$ can be deformed away from the singular point and from the results below one can see that this case does not lead to additional restrictions.

tube is small relative to l_s, it should be understood semi-classically as a 1-brane. To identify this brane, it is instructive to rewrite eq. (3.5) for the energy in the following form[12]

$$E = \frac{k}{\pi} T_1 \int d\rho \frac{\mathcal{A}^2}{g} = T_1 \int N'_T dr \,, \qquad N'_T = \frac{k}{\pi} \frac{\mathcal{A}^2}{|\mathcal{A}_0|} \,. \tag{4.10}$$

Comparing this to the energy of a (static) D1-brane, extended in the radial direction (at a fixed point in S^3_{6789})[13]

$$E = T_1 \int dr \,, \tag{4.11}$$

one sees that the above 1-brane can be identified as a collection of N'_T D1-branes. Such an identification requires that N'_T in eq. (4.10) should be an integer. To find the resulting condition on ψ_0, we consider $\psi_0 > 0$ and $z_\infty \to \infty$. This corresponds to $r_0 \to 0$ in eq. (3.4), so for a fixed r in the throat (satisfying $0 < r \ll \sqrt{k} l_s$), $\psi \to 0$, which implies

$$\mathcal{A} \approx \mathcal{A}_0 \approx -f \approx \psi_0 \,.$$

This gives

$$N'_T = \frac{k\psi_0}{\pi} \,, \tag{4.12}$$

leading to the integrality condition (4.9) with $N = N'_T$. The identification of the tube as a D1-brane was obtained here considering the tension of the tube. Alternatively, one can consider the D1 charge of the tube and compare it with that of a D1-brane. This was done in subsection 2.2. Eqs. (2.27), (2.28), (2.33) give

$$N'_Q = \frac{1}{(2\pi l_s)^2} \int_{S^2} \mathcal{F} = \frac{k}{\pi} f(\psi)$$

which, for $\psi \to 0$ coincides with eq. (4.12)[14].

Therefore, a distant observer sees N'_T coinciding D1-branes (extended in the $z = x^6$ direction) ending on a flat D3-brane (extended in the (x^7, x^8, x^9) directions). For $\psi_0 \le \pi$, the number of branes is at most k and all of them have finite extent, the other end being on the NS5-branes. For $\psi_0 > \pi$, the number N'_T of D1-branes above the NS5-branes ($z > 0$) is larger then k, but not all of them end on the NS5-branes, since the D3 tubes extends also to $z < 0$. Comparing the charge and tension of the tube above and below the NS5-branes (note the difference in the width of the tubes in figure 3), one finds exactly k D1-branes ending on the NS5-branes, while the other

[12]These expressions are derived using the BPS equation $z' = -f/g$, and the relation $rdr = (zz' + \rho)d\rho$.

[13]This expression is derived using the analog of eq. (2.20) for a D1-brane. Note that the factors of the harmonic function h cancel.

[14]More precisely, one obtains $N'_Q = -k\psi_0/\pi$. The minus sign means that the D1-branes are oriented in the *negative* direction of $z = x^6$.

$N'_T - k$ are semi-infinite, extending from the D3-brane to $z \to -\infty$. As will be seen in the next subsection, such a configuration can be obtained from a configuration with $N'_T = k$ by moving some of the NS5-branes down to $z \to -\infty$.

As the width of the tube is increased, both N'_T and N'_Q change continuously and are no longer integers. This is particularly bothering for N'_Q, which represents a RR charge. This apparent puzzle, observed in [6], was resolved in [8] (see also [10, 11]). Using the decomposition (2.22) with the choice (4.7) of B, one obtains

$$N'_Q = N_Q + \delta N_Q\,, \qquad N_Q = \frac{1}{2\pi}\int_{S^2} F\,, \qquad \delta N_Q = \frac{1}{(2\pi l_s)^2}\int_{S^2} B = \frac{1}{(2\pi l_s)^2}\int_{C_3} H\,,$$

where C_3 is the disc $[0,\psi]$ in S^3_{6789} (in which B is regular!). It was shown in [8] that, as the width of the tube is increased, there is a bulk contribution to the RR charge which exactly cancels δN_Q, so the total RR charge remains

$$N_Q = \frac{1}{2\pi}\int_{S^2} F = -\frac{k\psi_0}{\pi}\,, \tag{4.13}$$

which is integral and equal to the number of D1-branes ending on the D3 brane.

4.3 A dynamical explanation

Both the above derivations of the restriction (4.9) on the possible D3-brane configurations use indirect arguments and do not provide a real explanation for the restriction. Such an explanation will be given in this subsection. To obtain it, one separates the NS5-branes along the z axis and studies the resulting D3-brane configuration near $\rho = 0$. Considering all classical solutions with $0 < \psi_0 < \pi$ (i.e., ignoring the quantization restriction (4.9)), one finds two types of configurations:

1. the D3-brane passes between two NS5-branes;

2. the D3-brane intersects one of the NS5-branes.

Then, considering the integrality condition (4.9), one observes that it precisely distinguishes between these two types, allowing only the first one[15] This suggests the following dynamical origin for the quantization restriction:

> *The D3 and NS5-branes (in the relative orientation considered here) repel each-other and avoid intersecting one-another.*

Recall that this repulsion was explicitly observed in the solution with $\psi = \pi$ (figure 2). The only situation in which the two branes touch one-another is when NS5-branes from both sides of the D3-brane are brought to coincidence, trapping the D3 brane between them.

[15] This relation between the quantized ψ_0 and the possible positions of the D-brane among the NS5-branes was suggested independently in [23].

We now give some details of the analysis. The geometry induced by the above distribution of NS5-branes is still given by eqs. (2.7)-(2.9), but this time with the following harmonic function

$$h = 1 + \sum_i \frac{kl_s^2}{|\vec{y} - \vec{y}_i|^2} ,$$

where $\{\vec{y}_i = (z_i, 0, 0, 0)\}$ are the locations of the branes. The supersymmetry preserved by the geometry is still given by (2.19), since in its derivation, the explicit form (2.10) of h was not used. Likewise, in the derivation of the BPS equation (2.41), only the $SO(3)_{789}$ symmetry of h (and f) was used (as manifested by the use of cylindrical coordinates), so it remains valid also in the present case. The function f appearing in this equation is not that given in eq. (2.34), but a more general function of z, ρ. To obtain it, one recalls eqs. (2.33),(2.9):

$$kl_s^2 df \wedge \omega_2 = d\mathcal{F} = H = - *_4 dh ,$$

which imply a linear relation between df and dh and, since dh is a superposition of the contributions from each NS5-brane, so is df. Therefore, $kf = \sum_i f_i$, where f_i is obtained from the function f in eq. (2.34) by shifting it to the location of the i'th NS5-brane. In particular, f is locally constant along the z axis, and decreases discontinuously by $\frac{\pi}{k}$ at each of the locations of the NS5-branes.

To study the solutions of the BPS equation, we start with coinciding NS5-branes and a D3-brane configuration corresponding to $0 < \psi_0 < \pi$ and a large positive z_∞. We recall from the previous subsection that this configuration includes a thin 1-brane with tension

$$T = N_T' T_1 , \qquad N_T' = \frac{k\psi_0}{\pi} . \tag{4.14}$$

Now we separate the NS5-branes along the z axis until there is a large distance between them, so that in any given region, there is at most one NS5-brane that has a significant influence on the geometry. Thus, one can use the results of subsection 3.1 to study the corresponding D3-brane configuration. Changing the NS5-brane distribution means, technically, changing the functions h and f. Recall that f is defined up to an integration constant. We fix this ambiguity by requiring that the part of the D3-brane far above the NS5-branes remains unchanged. This is achieved by holding fixed the value $-\psi_0$ of f on the z axis, above all the NS5-branes and, consequently, the tension of the 1-brane above the NS5-branes is still given by (4.14). Now one follows this solution down, to the region of the NS5-branes. Near the upper NS5-brane there are the following possibilities:

- $(0 <) N_T' < 1$: the D3-brane intersects the NS5-brane, as in figure 4; the angle of approach is $N_T' \pi$.

- $N'_T = 1$: the tube closes just below this NS5-brane, as in figure 2, but above the other NS5-branes;

- $N'_T = 1$: the tube continues down below the NS5-brane, with a reduced tension, corresponding to N'_T replaced by $N'_T - 1$ in eq. (4.14). Note that this means that exactly one D1-brane ends on the NS5-brane.

In the last case, one can continue to follow the tube to the next NS5-brane and so on. Denoting

$$N'_T = \hat{N} - 1 + \frac{\hat{\psi}_0}{\pi}\,, \qquad (\hat{N} = 1, \ldots, k, 0 < \psi \leq \pi)\,,$$

one obtains that when N'_T is integer ($N'_T = \hat{N}$, $\hat{\psi}_0 = \pi$), the D3-brane passes between the $\hat{N}$th and $(\hat{N}+1)$th NS5 brane, while otherwise, the D3-brane intersects the $\hat{N}$th NS5-brane, with $\hat{\psi}_0$ being the approach angle. These are the results stated in the beginning of the subsection.

Strictly speaking, one should bear in mind that the throat of a single NS5-brane is highly curved, so to use semi-classical consideration, one should avoid these regions. This can be done by considering tubes that are wide enough. For such tubes, N'_T is not simply related to ψ_0, but one can consider, instead, N_Q (defined in eq. (4.13)), which is not modified by changing the width. Alternatively, one can restrict attention to NS5 distributions where there are several concentrations of branes, each with many branes, leading to a geometry which is weakly curved everywhere. Considering only such situations, one can show that configurations in which the D3-brane passes between NS5 branes always correspond to ψ_0 which satisfies the integrality condition.

4.4 Boundary conditions in the worldsheet CFT

So far, the analysis was semi-classical, the D3-brane being represented by a classical manifold in the NS5 geometry. In the throat region of this geometry ($r \ll \sqrt{k} l_s$), one can do better. The corresponding (supersymmetric) 2D non-linear σ model is an exactly solvable Conformal Field Theory (CFT) [2], so one can consider the full quantum dynamics of the fundamental string. This results with the same quantization conditions on k and ψ_0. We now review this approach and its relation to the previous ones.

In the worldsheet formulation of the string dynamics, a D-brane corresponds to a boundary of the worldsheet and different branes are distinguished by different boundary conditions imposed at the corresponding boundaries (see [24] and references therein). The boundary conditions studied so far for the throat background (2.13) have a factorized form (i.e. do not mix the various factors in (2.13)). Geometrically, this means that the intersection of the D-brane with the 3-sphere is independent of the other factors. This is the case only in a subset of the D3 brane configurations that were identified semiclassically in the previous section: ψ_0 is restricted to the range $(0, \pi)$ and for each such ψ_0, z_∞ has a specific value, corresponding to $r_0 \to \infty$ in eq. (3.4), so that in the throat the brane is described by $\psi = \psi_0$. Therefore, in the CFT approach one considers only this subset of D3-branes.

The S^3_{6789} factor in (2.13)) can be identified as the SU(2) group manifold, and the corresponding CFT is (the supersymmetric extension of) SU(2) WZW model. The $\mathrm{SO}(4)_{6789}$ symmetry of the geometry is manifested by the existence, in the CFT, of an $\widehat{\mathrm{su}}(2)$ affine algebra generated, in the left/right sectors, by the "currents" $J, \tilde{J}$ respectively.[16] The preservation of the $\mathrm{SO}(3)_{789}$ symmetry by the D-brane is achieved by imposing a "gluing condition" $J = \tilde{J}$, as part of the boundary condition. This gluing condition leads [20][17] (as required by the symmetry) to a worldvolume that intersects the 3-sphere at a 2-sphere with a fixed $\psi = \psi_0$. Imposing also the preservation of supersymmetry and following the approach in [13], one can show that the possible boundary conditions are in one-to-one correspondence with the primary states of the bosonic affine algebra[18] and there are $k - 1$ such states, labeled by their SU(2) spin j: $2j = 0, \ldots, k-2$. This agrees with the number of allowed 2-spheres implied by the quantization condition (4.9) (as observed in [20]), suggesting the identification

$$\psi_0 = N\frac{\pi}{k}, \qquad N = 2j+1 = 1, \ldots, k-1\,. \tag{4.15}$$

Evidence for this identification was found recently in [14], where the expectation values of bulk fields were calculated and their source (identified as the D-brane) was found to be concentrated around a 2-sphere with $\psi = \psi_0$ given by eq. (4.15). It is worth emphasizing that the analysis in [14] is performed in the framework of the *exact* CFT and is, therefore, valid also for k which is not large (although the D-brane is found to be delocalized in a band of width $\Delta\psi \sim 2\frac{\pi}{k}$, which is significant for small k).

Note that the values $\psi = 0, \pi$ ($N = 0, k$) are absent from the list (4.15), although they are allowed by the quantization condition (4.9). These values correspond to a D1-brane ending on the NS5-branes and, therefore, the absence of a corresponding boundary condition in the worldsheet description may seem disturbing. This puzzle is, however, resolved by the discussion in subsection 4.2. There it was demonstrated that a D3-brane with $\psi_0 = \pi/k$ (corresponding to $j = 0$ in eq. (4.15)), wrapped on a 2-sphere with a sufficiently small radius should be identified as a D1-brane. In the

[16]The level of this algebra is k and its representation theory implies that k is a (positive) integer, in agreement with the previous considerations.

[17]The geometrical meaning of gluing conditions as above was also discussed in [9, 12, 14], [22, 23], [25]–[29].

[18]The affine algebra acts both on the bosons and fermions in the CFT, with levels $k-2$ and 2 respectively.

JHEP08(2000)030

present case (i.e., in the throat; with $\psi = \psi_0$), the radius of this cylinder is

$$R_2 = l_s\sqrt{k}\sin\frac{\pi}{k} \lesssim \frac{\pi}{\sqrt{k}} l_s \lesssim l_s$$

(see eq. (2.36)), so it is indeed small enough.

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